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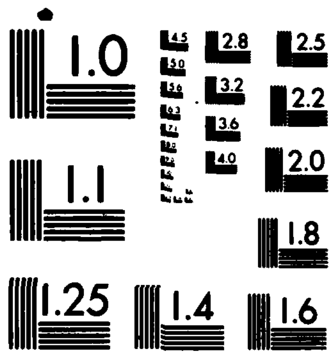

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During the year a number of publications appeared whose research was sponsored by the Grant. Copies of the first page of the papers are attached and will indicate the Journals in which the papers appeared and the abstracts will give an indication of the research contained in the papers. In addition to the research reported in the published papers referred to above, some progress was made in relating the inverse scattering problem to causality. In particular, Newton's "miracle" formula for the potential in three-dimensional inverse scattering was derived purely from causality considerations. Moreover, the one-dimensional analogue indicated some possible problems when point eigenvalues were present. Termination of the grant precluded completion of the paper and subsequent publication.

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TABLE OF CONTENTS

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2.0 RESEARCH DIRECTIONS

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APPENDIX A

PUBLICATIONS

"Eigenvalues and Eigenfunctions Associated with the Gel'fand-Levitan Equation," J. Math. Phys., 25, 1, 108, January 1984.

"Phases of Complex Functions from the Amplitudes of the Functions and the Amplitudes of the Fourier and Mellin Transforms," J. Opt. Soc. Amer., 73, 1451, November 1983.

"The Use of Comparison Filters in Linear Filter Theory," J. Math. Phys., 24, 11, 2550, November 1983.

Eigenvalues and eigenfunctions associated with the Gel'fand–Levitan equation

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(Received 8 June 1983; accepted for publication 10 August 1983)

It is shown here that the solutions of the Gel'fand–Levitan equation for inverse potential scattering on the line may be expressed in terms of the eigenvalues and eigenfunctions of certain associated operators of trace class. The details are sketched for the case of rational reflection coefficients, and carried out for the simplest class of examples.

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1. INTRODUCTION

The Gel'fand–Levitan equation plays a central role in solving inverse scattering problems in one dimension.[1] In the case where the problem involves a *scattering potential* $V(x)$ defined for $-\infty < x < +\infty$, for example, we know that $V(x)$ may be recovered from the *reflection coefficient* $r(k)$, defined for $-\infty < k < +\infty$, as follows: set

$$R(x,y) = \hat{r}(x+y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ikx} r(k) e^{-iky}\, dk, \tag{1}$$

and then solve for $K(x,y)$ the Gel'fand–Levitan equation

$$K(x,y) + R(x,y) + \int_{-\infty}^{x} K(x,z) R(z,y)\, dz = 0. \tag{2}$$

Then the potential $V(x)$ appears as

$$V(x) = 2\frac{d}{dx} K(x,x). \tag{3}$$

(See Ref. 2 for a general discussion of this procedure.)

In order to study the behavior of the solutions of (2), it is useful to consider the associated equation, to be solved for $K(x,y,w)$:

$$K(x,y,w) + R(x,y) + \int_{-\infty}^{w} K(x,z,w) R(z,y)\, dz = 0. \tag{4}$$

Evidently $K(x,y,x) = K(x,y)$. Now (4) may be expressed in operator form with w as a parameter:

$$K(w) + R + K(w)P(w)R = 0. \tag{5}$$

Here R, $K(w)$, and $P(w)$ are integral operators with kernels $R(x,y)$, $K(x,y,w)$, and $P(x,y,w)$, with

$$P(x,y,w) = \theta(w-x)\delta(x-y). \tag{6}$$

Here $\theta(z)$ is the Heaviside function, and $\delta(z)$ its derivative.

Now (4) yields

$$K(w)(I + P(w)R) = -R, \tag{7}$$

and hence, whenever $(I + P(w)R)$ is invertible,

$$K(w) = -R(I + P(w)R)^{-1}. \tag{8}$$

Now suppose that the reflection coefficient $r(k)$ is such that its Fourier transform $\hat{r}(z)$ is smooth and integrable. Then it follows that the operator $P(w)R$ is of *trace class* for each w, and

$$\operatorname{tr} P(w)R = \int_{-\infty}^{w} \hat{r}(2z)\, dz. \tag{9}$$

One can then define the *Fredholm determinant* $\Delta(w)$ of the operator $(I + P(w)R)$ by (cf. Ref. 3, p. 255ff)

$$\Delta(w) = \det(I + P(w)R) = \exp \operatorname{tr} \log(I + P(w)R). \tag{10}$$

Evidently

$$\log \Delta(w) = \operatorname{tr} \log(I + P(w)R) \tag{11}$$

and so

$$\frac{d}{dw}\log \Delta(w) = \frac{\Delta'(w)}{\Delta(w)} = \operatorname{tr} P'(w)R(I + P(w)R)^{-1} = -\operatorname{tr} P'(w)K(w). \tag{12}$$

Here we have used (8). But $P'(w)K(w)$ has kernel $\delta(w-x)K(x,y,w)$, so

$$-\operatorname{tr} P'(w)K(w) = -\int_{-\infty}^{w} \delta(w-x)K(x,x,w)\, dx = -K(w,w,w) = -K(w,w). \tag{13}$$

Hence by (3)

$$V(w) = 2\frac{d}{dw}K(w,w) = -2\frac{d^2}{dw^2}\log \Delta(w). \tag{14}$$

This formula, which gives V directly in terms of R, first appears in Ref. 4, and has since been rediscovered by several authors, including us.[5] In one sense, this formula by-passes the Gel'fand–Levitan equation, since it gives V directly in terms of R, and once V is known everything about the scattering problem is known, at least in principle.

In another sense (14) is no better than (4), since the calculation of the determinant $\Delta(w)$ of $(I + P(w)R)$ is not usually an easy matter in practice. One possible approach is to calcu-

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Phases of complex functions from the amplitudes of the functions and the amplitudes of the Fourier and Mellin transforms

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For the most part, the phase-retrieval problem has been dominated by the use of the Hilbert transform on the logarithm of the absolute value of the wave function. In our approach, by contrast, we use the intensity associated with the function and the intensities in Fourier- or Mellin-transform space corresponding to a set of apertures to obtain the phases within a constant by means of simple formulas. The set of apertures required is those arbitrarily close to a fixed aperture and its complement. The higher-dimensional cases are also treated for the case of the Fourier transform.

INTRODUCTION

The most commonly used method used to obtain the phase of a complex function $f(x)$ of a real variable x is to take the logarithm of $f(x)$, which in certain circumstances can be analytically continued in the complex plane. The Hilbert transform is then used to find the phase, which is the ratio of the imaginary part of the logarithm to the real part; the real part is the logarithm of the absolute value of the function. For unique results, the zeros of the analytic function must be prescribed. Various methods are used to find them from physical measurements, which correspond to the use of appropriate sets of apertures. A survey of approaches to the phase-retrieval problem is given in Ref. 1. A recent method for locating the zeros is given in Ref. 2.

As far as we can determine, the first departure from the use of the Hilbert transform was given in 1963 by Lomont and Moses.[3] The object of their approach was to give data in terms of intensities only. It was shown that a necessary and sufficient condition for two complex functions with the same amplitude to have the same phase within a constant was that the squares of the amplitudes in Fourier-transform space be equal for all apertures through which $f(x)$ is observed. In 1971 Gerchberg and Saxton[4] independently used the input required by the theorem of Ref. 3 to obtain the phase and gave numerical methods to obtain it. In Ref. 5 the intensities in Fourier-transform space are also used, together with analytic properties of the function and its transform, and, like the work of Ref. 4, that of Ref. 5 goes in a different direction from ours.

In this paper we give simple formulas for phase retrieval based on measurements using apertures arbitrarily close to a fixed aperture and its complement. We believe that our work is easier to use in many cases than that done earlier, although numerical implementation has yet to be done using our method. One of us (Moses) reviewed the material of Ref. 3 from a different point of view and showed also that the intensities in the Mellin-transform space could be used instead of those in the Fourier-transform space.[6]

Without going into detail, we report that the squares of the amplitudes of Fourier transforms correspond to intensities of the spectral components in $f(x)$ when the signal passes through gratings of varying aperture. Another well-known physical situation in which the function and its Fourier transform occur is that of optical systems, in which it can be shown that the wave functions in the aperture and the image plane constitute a Fourier-transform pair.[1]

The applications of phase retrieval are many and important. Among the applications are (dark-field) electron microscopy and holography. Other applications are mentioned in Refs. 1 and 5.

The significance of the Mellin transform in terms of scaling is discussed in Ref. 7, in which a method is also given whereby the intensities in the Mellin-transform space (in one dimension) can be obtained as a Fourier transform of a scale autocorrelation. The optical importance of the Mellin transform is discussed in Refs. 8 and 9, in which methods of implementing the transform optically are discussed.

Although the Mellin transform can be obtained from the Fourier transform by a change of variables, the two transforms represent different situations. In Ref. 7 it is shown that peaks in the Mellin-transform power spectrum correspond to features in the original function that are periodic in *magnification*. By contrast, peaks in the Fourier-transform power spectrum correspond to features that are periodic in *translation*. It thus behooves us to study phase retrieval in terms of the Mellin-transform power spectra as well as spectra of the Fourier transform.

The use of comparison filters in linear filter theory

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In the present paper, it is shown how the linear filter equation for a given correlation coefficient can be solved in terms of the solution of the filter equation with a different correlation coefficient. The second filter is called a comparison filter. One obtains an integral equation for the difference of the two filters in terms of the difference of the two correlation functions and the solution of the comparison filter. Thus if the comparison filter is known and its correlation coefficient is close to that of the desired filter, one may regard the comparison filter as being an approximation to it. The difference of the two filters is then small and perturbation expansions or variational principles for the difference may be expected to give better results than if one did not use a comparison filter. The difference in the solutions of the two filter equations may also be regarded as the change (or error) in the filter due to a change (or error) in the correlation coefficient. Our result is obtained by pressing the close analogy of the filter equation to the Gel'fand–Levitan equation of inverse spectral theory. Another result of the use of comparison filters is to show that the filter equation for the difference of filters satisfies a possibly useful grouplike property.

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I. INTRODUCTION

In Ref. 1, Kay and Moses treated the Gel'fand–Levitan equation of the inverse spectral theory problem from a very general point of view and observed that the Gel'fand–Levitan equation was a generalization of the filter equation of that time, namely, the Wiener–Hopf equation. This observation continues to hold for more general filters, for example, the Kalman filter. Recently, one of us (Moses, Ref. 2) gave a general scheme for introducing comparison potentials for which the solution of the corresponding Gel'fand–Levitan equation is known. The solution of any other Gel'fand–Levitan equation could be expressed in terms of the known solution through the use of an integral equation for the difference of the known and sought for Gel'fand–Levitan kernels. The use of comparison potentials led to perturbation schemes and variational principles which, in principle at least, led to more accurate approximations for the desired Gel'fand–Levitan kernel.

The purpose of the present paper is to give the analog for the filter equation. It is shown how the linear filter equation for a given correlation coefficient can be solved in terms of the solution of the filter equation with a different correlation coefficient. The second filter is called a comparison filter. One obtains an integral equation for the difference of the two filters in terms of the difference of the two correlation functions and the solution of the comparison filter. Thus if the comparison filter is known and its correlation coefficient is close to that of the desired filter, one may regard the comparison filter as being an approximation to it. The difference of the two filters is then small and perturbation expansions or variational principles for the difference may be expected to give better results than if one did not use a comparison filter.

The difference may also be regarded as the change (or error) in the filter due to a change (or error) in the correlati[on] coefficient. Another result of the use of a comparison filte[r is] to show that the filter equation for the difference of filter[s] satisfies a possibly useful grouplike property.

II. THE FILTER EQUATION AS A GEL'FAND–LEVITA[N] EQUATION

In dealing with the filter equation, we shall use standa[rd] notation as given, for example, in Kailath's monograph (R[ef.] 3). The filter equation is then

$$h(t,s) = K(t,s) - \int_{t_0}^{t} h(t,\tau)K(\tau,s)d\tau \qquad (t_0 < s < t < t_{f} \ldots$$

In Eq. (1), $h(t,s)$ is the filter matrix $h(t,s) = \{h_{ij}(t,s)\}$, and t[he] matrix $K(t,s) = \{K_{ij}(t,s)\}$ is related to the signal correlati[on] matrix $R_y(t,s)$ by

$$R_y(t,s) = I_p\delta(t-s) + K(t,s) \equiv Ey(t)y'(t).$$

The Gel'fand–Levitan equation, as treated in Ref. 1, is ide[n]tical to the filter equation in which $h(t,s)$ is the negative of t[he] Gel'fand–Levitan kernel $\langle t|K|s\rangle$, and the matrix $K(t,s)$ is t[he] driving kernel $\langle t|\Omega|s\rangle$ of Ref. 1. To press the analogy ev[en] further, it is useful to define the filter matrix as having th[e] triangularity property

$$h(t,s) = 0 \text{ for } s > t.$$

In the space of observables which include the signal[s] $y(t)$, let us define the operators in terms of integral operat[ors] with kernels. For example, if $f(t)$ is in the space, we shall wr[ite]

$$hf(t) = \int_{t_0}^{t} h(t,s)f(s)ds, \qquad R_y f(t) = \int_{t_0}^{t_f} R_y(t,s)f(s)d\ldots$$

and so on. Let us define the operator U by

$$Uf(t) = f(t) - hf(t).$$

In particular, if $y(t)$ is a signal,

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